NOTES

Two Points of Interpretation in Zeno

1. προέχειν in Zeno DK⁶ 29 B 1

The relevant part of the fragment is:

εἰ δὲ ἔστιν, ἀνάγκη ἕκαστον μέγεθός τι ἔχειν καὶ πάχος
καὶ ἀπέχειν αὐτοῦ τὸ ἕτερον ἀπὸ τοῦ ἑτέρου. καὶ περὶ τοῦ
προύχοντος ὁ αὐτὸς λόγος. καὶ γὰρ ἐκεῖνο ἕξει μέγεθος
καὶ προέξει αὐτοῦ τι. ὅμοιον δὴ τοῦτο ἅπαξ τε εἰπεῖν καὶ
ἀεὶ λέγειν· οὐδὲν γὰρ αὐτοῦ τοιοῦτον ἔσχατον ἔσται οὔτε
ἕτερον πρὸς ἕτερον οὐκ ἔσται.

I gave a simple explanation of this in two earlier arti-
cles.[1] I wish now to defend my view against all that has
been said on the subject since.[2] The crux of the matter is
the word προέχειν. Most other scholars[3] have taken this in
a physical sense: 'to jut out', 'to project', 'to be in the lead',
'to be the next one along (in a series of points)'. There are
several variations, but the fundamental criticism is the
same for all: the physical sense which they find in προέχειν
will not fit in with the physical sense that is to be found in
ἀπέχειν.

We have, to start with, an entity which has some size
and thickness. We can therefore find, in this entity, one
part which is separate[4] from another part. It is not made
explicit just what these two parts are. Are they two
end-points of the magnitude, or are they two halves of it,
or are they two pieces of it, taken from any part of the
extension? Again, what does 'separate' mean? Separate
with a gap between, or with no gap between? We are not
told; but perhaps for the purpose of this argument we do
not need to be told.

Whatever view we take of these uncertainties, the
word προέχειν taken in a physical sense will not do. For in
the second sentence the words τοῦ προύχοντος must refer
either to one or other or both of the two separate parts, or
else to some other part of the extension which is disjoint

[1] N. B. Booth, 'Zeno's Paradoxes', JHS lxxvii (1957) 199–200; 'Were
Zeno's Arguments a Reply to Attacks upon Parmenides?' Phronesis ii
(1957) 5–6.
[2] E.g. G. Vlastos, 'A Note on Zeno B1', in Allen and Furley (ed.),
Studies in Presocratic Philosophy (1975) 177–83; id., chapter on Zeno in
Philosophic Classics (ed. W. Kaufmann, 1961), 27–45; F. Solmsen, 'The
Tradition about Zeno of Elea Re-examined' in A. P. D. Mourelatos (ed.),
The Presocratics (1974); W. E. Abraham, 'The Nature of Zeno's Argument
Against Plurality in DK 29 B1', Phronesis xvii (1972) 46–52; D. J. Furley,
Two Studies in the Atomists (1967) ch. 5; E. Hussey, The Presocratics (1972)
101; M. Untersteiner, Zenone, Testimonianze e Frammenti (1963) 201.
[3] Earlier studies have included those of W. A. Heidel in Proc. Amer.
Acad. Arts and Sciences xlviii (1913), H. Fraenkel in Wege und Formen
fruehgriechischen Denkens (Munich 1955), H. D. P. Lee in Zeno of Elea
(Cambridge 1936).
[4] Vlastos takes ἀπέχειν to mean 'extend beyond', basing such an inter-
pretation on the use of the word on rare occasions in highly specialized
contexts (see references to his articles in n. 2). Objections to his interpre-
tation are:
(i) It is most implausible that Zeno would use ἀπ- and προ- to mean
virtually the same thing within so close range of each other; ἀπέχειν
should mean 'be separate from' unless there is something in the
context to bring out a different nuance.
(ii) It is really very difficult to obtain from the text, as Vlastos translates it,
the geometric sequence which he wants; each successive item, in his
interpretation, is first one thing, then it suddenly changes into
another.
(iii) The symmetric relation between the two ἕτερα is obscured or even
lost in his interpretation; 'right' and 'left' seem to matter in his
interpretation whereas in the text there is no such differentiation.
(This point is made by Abraham [loc. cit. 42–3].)

from either of the first two. If it refers to one or other of
the separate parts themselves, we are immediately in
difficulties. If we take the meaning to be 'the one in the
lead', why should either one or the other part be consi-
dered to be in the lead over the other?[5] If we say 'the
projecting part', it simply is not specially a projecting part
(or, for that matter, a jutting out part) in any reasonable
sense whatever; and why should one 'project' or 'extend'
from the other, rather than the other from it?

Let us therefore try the second alternative. Suppose the
words refer to some other part of the extension; possibly
(1) an outer layer of the extension, (2) a piece in-between
the first two parts, (3) a successive point, after the first two
points, in a line of points. The trouble with all these three
interpretations is that there is not enough in the Greek to
suggest them. If Zeno meant the piece in-between, he had
the word for 'in-between': he could easily have specified
that, if he had wanted. Any one who can find successive
'outer layers' in the passage is reading into the passage
matter which simply is not there. Likewise there is no
suggestion whatever that we are dealing with a line of
geometrical points;[6] any meaning of that kind would
have had to be made more explicit.

If προέχειν cannot be taken in a physical sense,[7] it
remains to ask in what sense it can be taken. One thing we
have noted is that neither one nor other of the two
separate parts, mentioned at the beginning of Zeno B 1,
has any preferential position with regard to the other. It is
therefore a fair presumption that τοῦ προύχοντος refers
equally to either one or other of these two separate parts.
If so, the choice of meaning is narrowed considerably. In
fact, the only possible meaning for τοῦ προύχοντος is the
'next' term in the sequence; that is, either one or other of
the two smaller units to be found in the first unit. This was
precisely the suggestion which I made in my earlier
articles, and it was for precisely this reason that I made it.

If so, the meaning of τοῦ προύχοντος is still 'the one in
the lead', that is, the meaning which is also found in the
Achilles Paradox. In the Achilles Paradox, the 'Slower' is
'in the lead', or 'ahead', in a race; in Zeno B 1, the next
term is 'in the lead', or 'ahead', in a sequence of terms. The
basic meaning is the same; the contexts make the differ-
ence in sense. It is scarcely to be expected that we should
find an exact parallel for the use of προέχειν in the sense of
one term 'succeeding' another, and I have not in fact
found one. Yet there is a parallel, in Aristotle himself, for
the prefix προ- used in the sense of progression in
sequence; in the Nicomachean Ethics (1094a20), Aristotle
rejects the notion of an infinite succession of 'ends', using
the words 'πρόεισι . . . εἰς ἄπειρον'. It is therefore highly
plausible that προέχειν should mean to be 'in the lead' (or

[5] Those who argue for the meaning 'in the lead' base their case strongly
on the Achilles argument (Phys. 239b17–19), in which προέχειν is used of
the 'Slower' still being a bit ahead at every stage. But the meaning there is
to 'be ahead in a race', and it applies only accidentally to successive points
in a geometric progression.
[6] As is suggested by H. D. P. Lee (loc. cit.) who translates τοῦ προύχοντος
correctly as 'successor' (or 'the one beyond'), but takes this in the physical
sense of points going along a line, each one 'beyond' its predecessor. How
near he was!
[7] One might note here that E. Hussey takes τοῦ προύχοντος in the
non-physical sense of 'the former', which is dubious Greek and, in my
view, makes little sense.

'ahead'), in the sense 'farther on in sequence', in Zeno B 1. Further, the only natural way, to my mind, of interpreting πρό in Simplicius 139, 16–18 (πρὸ τοῦ λαμβανομένου ἀεί τι εἶναι διὰ τὴν ἐπ' ἄπειρον τομήν) is 'beyond', in the sense of being 'farther on' in the process of dichotomy.

If προέχειν is taken in this sense, there is no longer any difficulty in interpreting the fragment; everything falls easily into place. In any magnitude, there must be one part separate from another. Now take the 'next' one, that is, either one or other of the two separate parts. This is the next term in the sequence. It, too, will have magnitude, and therefore, by the very same argument, there will be another term beyond it (αὐτοῦ in προέξει αὐτοῦ τι need not be partitive, in spite of Heidel, Fraenkel and Vlastos). The only difficulty that remains lies in 'ἕτερον πρὸς ἕτερον' in the last sentence; this might possibly be emended (as has already been suggested) to ἕτερον πρὸ ἑτέρου. The words are not easy to interpret, as they stand, under any of the other theories about the meaning of the whole fragment.

The detail of this interpretation must have a powerful effect upon our view of the general purport of Zeno's argument. First, interpreting προέχειν as I do, I take it that the first stage division is into two parts, each of which is then divided into two further parts, and so on, so that at the nth stage we have 2^n products of division. This implies a flat contradiction of the geometric series proposed by Vlastos (loc. cit.), and it brings me much closer to Abraham (loc. cit.).

A point of major philosophical and mathematical importance is as follows: if the division is into 2^n parts at every stage, then supposing n could actually reach infinity and we could thus obtain a so-called 'through-and-through' division, we should arrive at an infinite set of limit points which would have the order of the continuum. Abraham takes up this suggestion, and appears to infer in B 1 just such a through-and-through division. I cannot accept this. It is essential to Zeno's argument that, at every stage in the process of division, the products of the division all exist and have magnitude; otherwise he could not claim that there is no last term. There is therefore no division away into points having no magnitude. Further, since there is no last term, we never come to the end of the dividing process, and therefore a 'through-and-through' division is excluded. Those who wish to find an ancient discussion of 'through-and-through' division should refer to Aristotle De Generatione et Corruptione 315 b 25 ff., not to Zeno Fr. B 1. It is quite wrong to suppose that Zeno entertained any such idea in this fragment.

2. The fourth argument against motion

D. J. Furley, in Two Studies in the Greek Atomists (1967) ch. 5, suggests a new interpretation for the fourth paradox on motion. The relevant Greek text is Aristotle Phys. 240a9 ff. Here Furley picks out the words ἴσον γὰρ ἐκάτερόν ἐστι παρ' ἕκαστον and ἴσον χρόνον παρ' ἕκαστον γιγνόμενον τῶν Β ὅσον περ τῶν Α, ὥς φησι as being of particular importance. He thinks, on this basis, that: 'The essential step is that since a B and a Γ are opposite to the same A for the same time, they must be opposite to each other for that time; and this is fallacious'. He adds that Zeno here used static vocabulary (ἐστι, γίγνεσθαι παρὰ ἕκαστον) and that this was the source of his error. If so, this would still convict Zeno of error, but the error would be much less glaring and much more understandable than

the error about relative motion attributed to him by Aristotle.

Furley's interpretation of the Greek is open to criticism. The second passage he stresses (ἴσον χρόνον . . . τῶν Α) is cut out as a gloss by W. D. Ross. Ross saw, (1) that the participle here succeeds not just τὸ πρῶτον Γ (as it ought to, if the words are to make sense), but τὸ πρῶτον Γ καὶ τὸ πρῶτον B, and (2) that the words simply do not fit into the general sequence of thought of the sentence as a whole. He discussed the passage clearly in Aristotle's Physics (1936) p. 665. It will be seen, from his discussion, that the whole passage abounds in textual doubts and in difficulties of detailed interpretation. I do not claim that Ross or any one else has resolved all these difficulties beyond doubt, but Ross produced powerful arguments which Furley has not met.

Ross is criticized by M. C. Stokes[8] on the grounds that, if the 'offending' passage is excised, then the final words of the sentence, 'διὰ τὸ ἀμφότερα ἴσον χρόνον παρὰ τὰ Α γίγνεσθαι', are left in the air, with no proper logical consequence. On the contrary, their logical consequence is that the leading B and the leading Γ must arrive simultaneously at their final position, which is precisely what is stated in the preceding sentence.

Further, it is possible that Furley is overstressing the static nature of the vocabulary γίγνεσθαι and ἐστι. Surely it is a perfectly natural use of the Greek language to use these words to describe the time during which one onkos passes another one? So even if Ross is wrong in excising the 'offending' passage—though he certainly has strong reasons for wishing to do so—Furley's argument is still far from conclusive.

My own explanation of Zeno's error was that, at this early time, people were not familiar with the idea of relative motion. Furley criticized this explanation as inadequate; he says that the Achilles argument itself shows that Zeno had thought about relative motion. I must confess that I am not convinced by this argument of Furley's. The Achilles argument concerns the impossibility of Achilles overtaking the 'pursued'. It says nothing whatever about the basic problem of relative motion which is brought up in the fourth paradox, namely, the differentiation between motion relative to a 'fixed' frame of reference (if indeed such a 'fixed' frame exists!) and motion relative to a moving frame of reference. We now learn to make such a differentiation in Mathematics lessons at school (often not without difficulty, in our younger days). We have to think back to an earlier time when such a differentiation was by no means 'common knowledge'.

N. B. Booth

Department of Mathematics,
Polytechnic of North London

[8] One and Many in Presocratic Philosophy (1971) 185 n. 32. It would have been easier for Furley's interpretation if 'παρὰ ἕκαστον τῶν Α' had been written here instead of 'παρὰ τὰ Α'; as it is, the plural may indicate that the meaning is 'since both spend an equal time in going past all the A's'.

Etruscan graffiti on Oxford 213

The very full recent publication of the Athenian black figure amphora, Ashmolean Museum 213,[1] invites reflec-

[1] CVA Oxford iii, pls. 14; 15.1, 2.